Comparison of Constant Lift-Coefficient Climbs with Flight-Manual Climbs for a Jet Transport

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Aircraft climb trajectories utilizing a constant lift coefficient are investigated for the KC-135. Analytical and numerical approaches are presented. The objective is to climb to a prespecified altitude with a given amount of fuel and maximize the range. The optimum constant C_L is determined and the corresponding trajectory compared with that resulting from flight-manual climbs. Although the constant C_L strategy is easy to implement, its resulting efficiency is not as great as that resulting from the flight-manual climb.

Nomenclature

X	= surface range
h	= altitude
V	= velocity
γ	= flight-path angle with respect to horizontal
m	= total mass
g	= acceleration due to gravity
D	= $\frac{1}{2}\rho SV^2C_D$ = drag force
L	= $\frac{1}{2}\rho SV^2C_L$ = lift force
ρ	= air density
S	= reference area
$T(h,v)$	= thrust
ϵ	= thrust angle of attack
α	= angle of attack
C_L	= lift coefficient
C_D	= drag coefficient
C_{D0}	= drag coefficient at zero lift
$\mu(h,v)$	= specific fuel consumption
$V^*(t_i)$	= specified cruise velocity
$\dot{m}^*(t_i)$	= fuel consumption rate during cruise
AR	= aspect ratio
e	= efficiency factor
k	= $(\pi ARe)^{-1} = \alpha/C_L$

I. Introduction

VARIOUS people from time to time have suggested that flying a constant lift coefficient (C_L) during enroute climb would be an efficient control strategy. Such a strategy would certainly be easy to implement. A recent paper by Andrews[1] suggested that this strategy would yield a trajectory approximating that of a minimum time-to-climb strategy. Raybould[2] used an energy height approach to show that, in order to maximize range during climb, one should fly a C_L that maximizes the lift-to-drag ratio. It can be shown that the C_L that maximizes the lift-to-drag ratio is a constant. In this paper, the problem of determining the optimum constant C_L for a KC-135 is investigated analytically and through simulation and the solution compared to the flight-manual climb. The objective is to control C_L to maximize range during climb for a fixed amount of fuel. Final altitude is specified. A standard atmosphere is assumed and normal rated thrust is maintained at all times.

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II. Analytical Approach

The following derivation determines that value for C_L which instantaneously maximizes the incremental increase in range for an incremental amount of fuel used. Figure 1 denotes the coordinates and variables to be used in the analysis. The equations of motion are

$$\dot{x} = V \cos\gamma \tag{1}$$

$$\dot{h} = V \sin\gamma \tag{2}$$

$$\dot{v} = -g \sin\gamma - m^{-1}(D - T\cos\alpha) \tag{3}$$

$$\dot{\gamma} = V^{-1}[-g\cos\gamma + m^{-1}(L + T\sin\alpha)] \tag{4}$$

$$\dot{m} = -(\mu T/g) \tag{5}$$

$$\alpha = KC_L \tag{6}$$

The assumption is made that the flight-path angle γ is small ($\cos\gamma \approx 1$) and that its inertia is very small ($\dot{\gamma}V \ll 1$), so that lift is approximately equal to weight, i.e.,

$$L - W = 0 \tag{7}$$

The equation of lift is given by

$$L = \tfrac{1}{2}\rho SC_LV^2 \tag{8}$$

Combining (7) and (8) and solving for velocity yields

$$V = [2W/\rho SC_L]^{1/2} \tag{9}$$

Since the flight-path angle is small, Eqs. (1) and (5) can be rewritten as

$$\dot{X} = V \tag{10}$$

and

$$\dot{W} = -\mu T \tag{11}$$

In order to maximize the amount of range attained for an incremental amount of fuel used, consider the ratio of (10) and (11)

$$dX/-dW = V/\mu T \tag{12}$$

This is the quantity that must be maximized with respect to C_L. Because of the assumption that the inertia terms are negligible, the ratio of thrust to weight is given by

$$T/W = D/L = C_D/C_L \tag{13}$$

or

$$T = (C_D/C_L)W \tag{14}$$

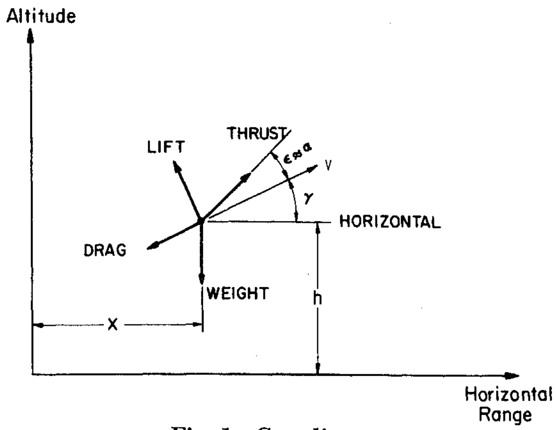


Fig. 1 Coordinates.

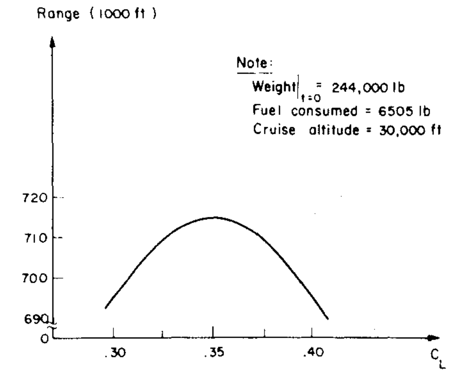


Fig. 2 Range achieved for constant C_L.

Substituting (9) and (14) into (12) yields

$$dX/-dW = (\mu C_D)^{-1}[2C_L/\rho SW]^{1/2} \tag{15}$$

The drag coefficient is assumed to be

$$C_D = C_{D0} + KC_L^2 \tag{16}$$

Substituting (16) into (15) yields

$$dX/-dW = [\mu(C_{D0} + KC_L^2)]^{-1}[2C_L/\rho SW]^{1/2} \tag{17}$$

Rather than maximize this quantity, it is easier to minimize the inverse with respect to C_L. The requirement for a minimum is

$$\frac{\partial}{\partial C_L}\frac{-dW}{dX} = \mu\left[\frac{\rho SWC_L}{2}\right]^{1/2}\cdot\left[\frac{3}{2}K - \frac{1}{2}\frac{C_{D0}}{C_L^2}\right] = 0 \tag{18}$$

or

$$C_L = [C_{D0}/3K]^{1/2} \tag{19}$$

Using the KC-135 data, $C_{D0} = 0.015$ and $K = 0.042$ yields $C_L = 0.35$.

Raybould[2] states that range is maximized during climb if the ratio of lift to drag is kept maximum. This ratio can be shown to be at a maximum when

$$C_L = (C_{D0}/K)^{1/2} = 0.597 \tag{20}$$

These two values are certainly candidates for the C_L that will maximize the range during the enroute climb. In order actually to determine the best value of C_L, a spectrum of values was used in conjunction with a digital-computer simulation of the nonlinear differential equations of motion.

III. Numerical Approach

As stated earlier, final altitude and fuel are specified. Since the control strategies may cause the desired altitude to be reached before all of the specified fuel is utilized, some means of accounting for this must be introduced. The following performance index is used:

$$1P = X(t_i) + [V^*(t_i)/-\dot{m}^*(t_i)][m(t_i) - m(t_f)] \tag{21}$$

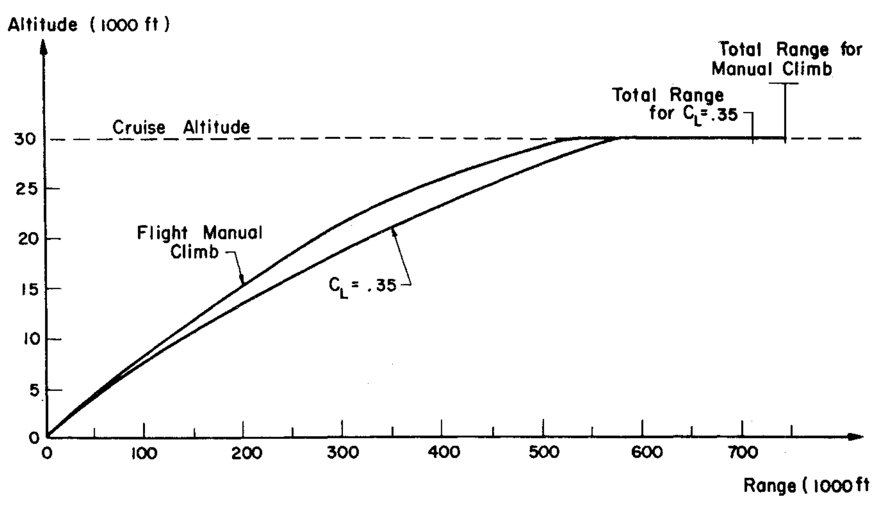
The first term is the range achieved at time t_i, the time at which the specified altitude is reached. The second term represents the additional range that can be achieved using the remaining fuel and flying at the velocity prescribed by the flight manual. The final mass $m(t_f)$ is specified, i.e., this is a fixed-fuel maximum-range problem.

Using this performance index, a comparison of the trajectories resulting from different values of C_L can be made and the best constant C_L determined. After finding this value and the corresponding performance, a comparison can be made to the flight-manual climb strategy. The flight manual instructs the pilot to fly at 280 knots calibrated airspeed until an altitude of 32,500 ft is reached and then at a Mach number of 0.78 until cruise altitude is reached.

The aircraft performance is rather insensitive to small initial changes in position X, velocity V, and flight path angle γ. There is no reason to consider any variation in initial height, since a constant height can be used to define the initial condition. Therefore, only changes in initial mass were considered. The following are the initial conditions that were held constant:

$$X(t_0) = 0 \text{ ft} \qquad h(t_0) = 300 \text{ ft}$$
$$V(t_0) = 483 \text{ fps} \qquad \gamma(t_0) = 0.13 \text{ rad} \tag{22}$$

Fig. 3 Trajectories for C_L = 0.35 climb and for flight-manual climb.



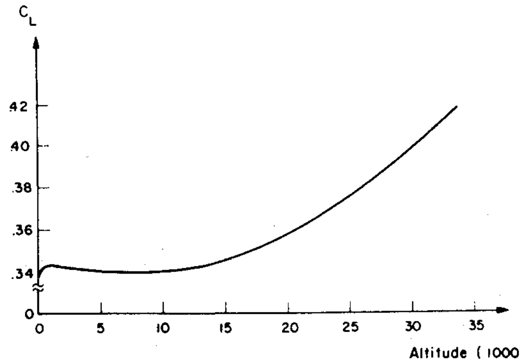


Fig. 4 Control variable for flight-manual climb.

The trajectories were broken into two segments, the climb phase and the cruise phase. Then the index of performance, Eq. (21), was used to find the best-constant C_L for that particular initial weight.

The first initial weight to be considered was 244,000 lb. Initial weights greater than 200,000 lb are of most interest. Savings in fuel consumption should be more significant for these heavier loads. Trajectories were found for C_L equal to 0.30, 0.35, 0.375, 0.40, and 0.597. Also, the trajectory for the flight-manual climb was found. Table 1 gives the range attained for 6505 lb of fuel consumed and a cruise altitude of 30,000 ft. Figure 2 is a plot of these results and shows that the peak is rather flat. Maximum range was achieved with a C_L of about 0.35. This agrees with the analytic results. However, as can be seen in Table 1, the range obtained for the flight-manual climb is substantially greater than for the best-constant C_L. Figure 3 shows the trajectories for the flight-manual climb and a constant C_L (0.35) climb. Even though the flight-manual climb has a lower C_L for the first 18,000 ft of altitude than does the $C_L = 0.35$ climb, the flight manual climb is able to maintain a higher trajectory because of increased lift due to its higher velocity. The flight-manual climb reaches the cruise altitude in a shorter distance than does the constant C_L climb. The reason for this can be seen by observing the control variable, Fig. 4. The flight-manual climb has the pilot initially fly a low C_L in order to build up velocity and then increase C_L for a steeper climb.

Table 1 Range attained using 6505 lb of fuel[a]

Type of control	Range (nearest 1000 ft) Climb phase	Cruise phase	Total
$C_L = 0.300$	695,000	0	695,000
$C_L = 0.350$	575,000	140,000	715,000
$C_L = 0.375$	540,000	170,000	710,000
$C_L = 0.400$	516,000	178,000	694,000
Flight-manual climb	522,000	226,000	748,000

[a] Initial weight = 244,000 lb, cruise altitude = 30,000 ft.

Table 2 Comparison of best-constant C_L climb to flight-manual climb

Weight $t = 0$, lb	Control strategy	Cruise altitude, ft	Fuel consumed lb	Range attained (nearest 1000 ft)
160,000	$C_L = 0.35$	32,000	3512.7	281,000
160,000	flt.-man. climb	32,000	3512.7	347,000
200,000	$C_L = 0.35$	32,000	4733.4	432,000
200,000	flt.-man. climb	32,000	4733.4	481,000
244,000	$C_L = 0.35$	30,000	6505.2	715,000
240,000	flt.-man. climb	30,000	6505.2	748,000
300,000	$C_L = 0.35$	27,000	7125.9	778,000
300,000	flt.-man. climb	27,000	7125.9	774,000

The same procedure was used to determine the best-constant C_L for initial aircraft weights of 160,000, 200,000, and 300,000 lb. A C_L slightly higher than 0.35 is best for an initial weight of 300,000 lb, and a C_L slightly lower than 0.35 is best for an initial weight of 160,000 lb. The best-constant C_L to use, for a wide range of initial weights, is 0.35. Of all the initial weights used, only for 300,000 lb is the constant C_L climb superior to the flight-manual climb (see Table 2).

The results of this investigation show that the flight-manual climb is superior to the constant C_L climb when using maximum range for a fixed amount of fuel as a criterion for comparison.

References

[1] Andrews, R. H., "Optimum Climb Trajectories at Constant Lift Coefficient," *Journal of Aircraft*, Vol. 6, No. 5, Sept.-Oct. 1969, pp. 475–477.

[2] Raybould, B., "Approximate Trajectories for Maximum Range," *Journal of the Royal Aeronautical Society*, Vol. 67, June 1963, pp. 385–386.